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ADDENDUM TO "BEHNKE-STEIN THEOREM FOR ANALYTIC SPACES"

BY

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ABSTRACT. A very simple argument shows that Theorem 3.1 in my paper *Behnke-Stein theorem for analytic spaces*, (these Transactions, 199 (1974), pp. 317 326) is enough, via a Narasimhan result, to obtain information about the torsion of the homology groups of a Runge pair of Stein spaces.

Let (X, Y), with $Y \subset X$, Y open, be a pair of reduced complex analytic spaces of (complex) dimension n. Andreotti and Narasimhan proved in [1], among many others, the following result:

(1.1) If (X, Y) is a Runge pair of Stein spaces (a 1-Runge pair in the terminology of [3]) and every singularity of X outside Y is isolated, then

$$H_r(X \bmod Y, \mathbf{Z}) = 0$$

for $r \geq n + 1$.

We wish to show the following statements (1.2) and (1.3),

(1.2) (Narasimhan [2, Theorem 3]): if X is a Stein space, then:

$$H_r(X, \mathbf{Z}) = 0 \quad \text{for } r \geq n + 1$$

and

$$H_n(X, \mathbf{Z}) \text{ is without torsion;}$$

(1.3) (Silva [3, Theorem 3.1]): if (X, Y) is a Runge pair of Stein spaces (or, equivalently, in the terminology of [3], a 1-Runge pair of cohomologically 1-complete spaces) then

$$H_{n+1}(X \bmod Y, \mathbf{C}) = 0;$$

make us able to remove from (1.1) the assumption that the singularities of X outside Y are isolated.

Indeed, if we write the exact homology sequence for the pair (X, Y):

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$$\cdots \longrightarrow H_r(X, \mathbf{Z}) \longrightarrow H_r(X \bmod Y, \mathbf{Z}) \xrightarrow{\varphi_r} H_{r-1}(Y, \mathbf{Z}) \longrightarrow \cdots$$

from (1.2) we obtain that $H_r(X \bmod Y, \mathbf{Z}) = 0$ for $r > n + 1$. Suppose now $r = n + 1$. (1.3) implies that $H_{n+1}(X \bmod Y, \mathbf{Z})$ is a torsion group. If we look again at the exact homology sequence for (X, Y) we see that the morphism:

$$\varphi_{n+1}: H_{n+1}(X \bmod Y, \mathbf{Z}) \longrightarrow H_n(Y, \mathbf{Z})$$

is injective, so that, $H_n(Y, \mathbf{Z})$ being without torsion, we must have $H_{n+1}(X \bmod Y, \mathbf{Z}) = 0$.

In conclusion we have obtained the following

THEOREM. *If (X, Y) is a Runge pair of Stein spaces, then*

$$H_r(X \bmod Y, \mathbf{Z}) = 0,$$

for $r \geqslant n + 1$.

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